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Review

Some new solutions of generalized Drinfel'd–Sokolov–Wilson system using Exp-function method

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In this paper, Exp-function method was used to construct solitary solutions of the generalized Drinfel'd–Sokolov–Wilson (DSW) system. It is shown that the Exp-function method, with the help of symbolic computation, provides a straightforward and powerful mathematical tool to solve nonlinear evolution equations with higher order nonlinearity. It is observed that the suggested scheme is highly reliable and may be extended to other nonlinear differential equations.

Key words: Exp-function method, solitary solution, periodic solution, generalized Drinfel'd-Sokolov-Wilson (DSW) system.

INTRODUCTION

The investigation of travelling wave solutions of nonlinear evolution equations plays an important role in the study and the mathematical modeling of diversified physical phenomena. Finding exact solutions of nonlinear evolution equations (NLEEs) has become one of the most exciting and extremely active areas of research investigation. The investigation of exact travelling wave solutions to nonlinear evolution equations plays an important role in the study of non-linear physical phenomena.

Recently many new approaches to nonlinear evolution equations have been proposed, for example, the variational iteration method (He, 1999; Noor et al., 2008, 2009), the tanh-method (Wazwaz, 2005), the homogeneous balance method (Wang, 1996; Fan and Zhang, 1998; Xiqiang et al., 2006), the F-expansion method (Fan and Jian, 2002; Zhang et al., 2006), the sine–cosine method (Wazwaz, 2004), the extended Fan's sub-equation method (Yomba, 2005), the simplest equation method (Kudryashov, 2005), the (G/G)-expansion method (Wang et al., 2008) and so on.

Also a new method called the Exp-function method was first proposed by He and Wu (2006) and was successfully applied to a KdV equation with variable coefficients (Zhang, 2007), high-dimensional nonlinear evolution equations (Noor et al., 2010), generalized equations with higher order nonlinearity (Zhou et al., 2008; Misirli and Gurefe, 2010; Gomez and Salas, 2010), differential-difference equations (Bekir, 2010; Noor et al., 2008), system of nonlinear partial differential equation (Noor et al., 2009), stochastic equation (Dai and Zhang, 2009), etc. On the other hand, the Exp-function method was extended to construct multi-wave solutions to nonlinear evolution equations in Noor et al. (2012) and Dai et al. (2010).

In this paper, the Exp-function method was applied to the generalized Drinfel'd–Sokolov–Wilson system given in Wazwaz (2006) and Sweet and Gorder (2010) as:

$$u_t + \left(v^n\right)_x = 0, \qquad (1)$$

$$v_t - av_{xxx} + 3bu_x v + 3duv_x = 0. \qquad (2)$$

EXP-FUNCTION METHOD

Considering the general nonlinear partial differential equation of the type, we have the following equation:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xxx}, \ldots) = 0. \qquad (3)$$

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Using a transformation, the following is obtained:

$$\eta = kx - \omega t, \tag{4}$$

Where k and ω are constants, we can rewrite equation (3) in the following nonlinear ODE:

$$Q(u, u', u'', u''', u^{iv}, \ldots) = 0. \tag{5}$$

Where the prime denotes derivative with respect to η.

According to the exp-function method, which was developed by He and Wu (2006), we assume that the wave solutions can be expressed in the following form:

$$u(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp[n\eta]}{\sum_{m=-p}^{q} b_m \exp[m\eta]}, \tag{6}$$

Where p, q, c and d are positive integers which are known to be further determined, and a_n and b_m are unknown constants. We can rewrite equation (6) in the following equivalent form:

$$u(\eta) = \frac{a_c \exp(c\eta) + \ldots + a_{-d} \exp(-d\eta)}{b_p \exp(p\eta) + \ldots + b_{-q} \exp(-q\eta)}. \tag{7}$$

To determine the value of c and p, the linear term of the highest order of equation (6) was balanced with the highest order nonlinear term. Similarly, to determine the value of d and q, the linear term of the lowest order of equation (5) was balanced with the lowest order non linear term.

SOLUTION PROCEDURE

Here, the exp-function method was applied for the generalized Drinfel'd–Sokolov–Wilson (DSW) system.

GENERALIZED DRINFEL'D–SOKOLOV–WILSON SYSTEM

This study considered the generalized Drinfel'd–Sokolov–Wilson (DSW) system:

$$u_t + (v^n)_x = 0,$$

$$v_t - av_{xxx} + 3bu_x v + 3duv_x = 0.$$

Introducing a transformation as $\eta = kx - \omega t,$ Drinfel'd–Sokolov–Wilson (DSW) system can be converted into ordinary differential equations:

$$-\omega u' + (v^n)' = 0, \tag{8}$$

$$-\omega v' - ak^2 v''' + 3bu'v + 3duv' = 0. \tag{9}$$

Where the prime denotes the derivative with respect to η.

Integrating equation (8), we obtained:

$$u = \frac{v^n + c}{\omega} \tag{10}$$

Where c is an integration constant.

Substituting u into equation (9) yields:

$$a\omega(n+1)(n+2)k^2(v')^2 - 6(bn+d)v^{n+2} + (n+1)(n+2)(\omega^2 - 3cd)v^2 = 0 \tag{11}$$

Using the transformation:

$$v = \phi^{\frac{1}{n}} \tag{12}$$

Equation (11) becomes:

$$a\omega(n+1)(n+2)k^2(\phi')^2 - 6n^2(bn+d)\phi^3 + n^2(n+1)(n+2)(\omega^2 - 3cd)\phi^2 = 0 \tag{13}$$

The trial solution of equation (13) can be expressed as follows:

$$\phi(\eta) = \frac{a_c \exp(c\eta) + \ldots + a_{-d} \exp(-d\eta)}{b_p \exp(p\eta) + \ldots + b_{-q} \exp(-q\eta)}. \tag{14}$$

To determine the value of c and p, the linear term of the highest order of equation (13) was balanced with the highest order nonlinear term. Proceeding as before, we obtained

$p = c$ and $d = q$.

Case 3.1.1

We can freely choose the values of c and d, but it will be illustrated that the final solution does not strongly depend upon the choice of values of c and d. For simplicity, we set $p = c = 1$ and $q = d = 1$; thus equation (14) reduces to:

$$\phi(\eta) = \frac{a_1 \exp[\eta] + a_0 + a_{-1} \exp[-\eta]}{b_1 \exp[\eta] + a_0 + b_{-1} \exp[-\eta]}. \tag{15}$$

Substituting equation (15) into equation (13), we have:

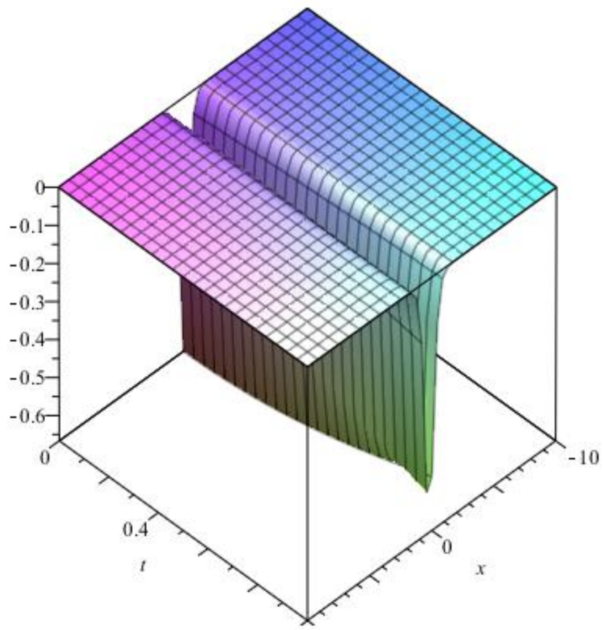

Figure 1. Graph of $v(x,t)$ for $b_{-1}=1, b_0=0.2, k=3, a=0.1, b=1, \omega=2, d=1, c=1, n=1$.

$$\frac{1}{A}\left[\begin{array}{l} c_4\exp(4\eta)=c_3\exp(3\eta)+c_2\exp(2\eta)+c_1\exp(\eta)+c_0+c_{-1}\exp(-\eta)+c_{-2}\exp(-2\eta) \\ +c_{-3}\exp(-3\eta)+c_{-4}\exp(-4\eta) \end{array}\right]=0, \quad (16)$$

Where $A=\left(b_1\exp(\eta)+b_0+b_{-1}\exp(-\eta)\right)^4, c_i$ are constants obtained by Maple 17. Equating the coefficients of $\exp(n\eta)$ to be zero, we obtained:

$$\{c_{-4}=0, c_{-3}=0, c_{-2}=0, c_{-1}=0, c_0=0, c_1=0, c_2=0, c_3=0, c_4=0\}. \quad (17)$$

For the solution of equation (17), we have the following solution set which satisfies the given generalized Drinfel'd–Sokolov–Wilson system:

1st solution set

$$\left\{\begin{array}{l} k=\dfrac{-\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}, a_{-1}=0, a_0=\dfrac{-b_0(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)}{3(bn+d)} \\ a_1=0, b_{-1}=b_{-1}, b_0=b_0, b_1=\dfrac{1}{4}\dfrac{b_0^2}{b_{-1}} \end{array}\right\}$$

We therefore obtained the following generalized solitary solution (Figure 1):

$$\phi(x,t)=\frac{-\frac{1}{3}\left(b_0\left(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2\right)\right)}{\left((bn+d)\left(b_{-1}e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}+b_0+\dfrac{b_0^2}{4b_{-1}}e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}\right)\right)}$$

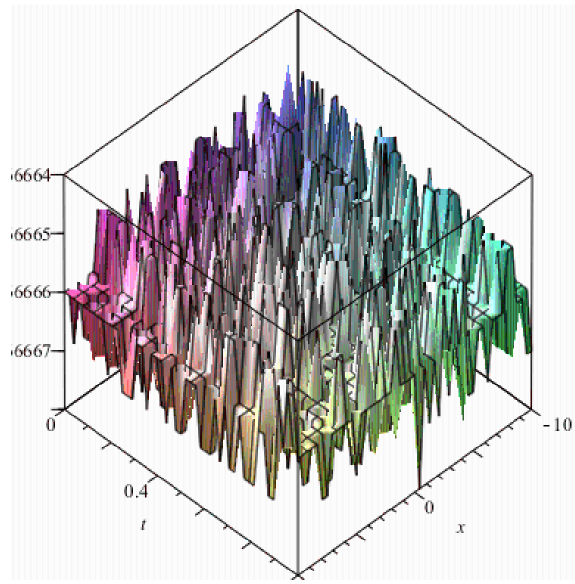

Figure 2. Graph of $v(x,t)$ for $b_1=1, b_{-1}=1, b_0=2, k=3, a=1, b=2, \omega=1, d=1, c=1, n=1$.

$$v(x,t)=\left(\frac{-\frac{1}{3}\left(b_0\left(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2\right)\right)}{\left((bn+d)\left(b_{-1}e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}+b_0+\frac{1}{4}\dfrac{b_0^2}{b_{-1}}e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}\right)\right)}\right)^{1/n}$$

2nd solution set

$$\left\{\begin{array}{l} k=k, a_{-1}=\dfrac{-b_{-1}(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)}{6(bn+d)}, a_0= \\ \dfrac{-b_0(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)}{6(bn+d)}, a_1=\dfrac{-b_1(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)}{6(bn+d)}, \\ b_{-1}=b_{-1}, b_0=b_0, b_1=b_1 \end{array}\right\}$$

We therefore obtained the following generalized solitary solution (Figure 2):

$$\phi(x,t)=\frac{\left(\begin{array}{l} \dfrac{-(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)b_{-1}}{6(nb+d)}e^{-xk+\omega t}-\dfrac{(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)b_0}{6(nb+d)}- \\ \dfrac{b_1(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)}{6(nb+d)}e^{xk+\omega t} \end{array}\right)}{\left(b_{-1}e^{-xk+\omega t}+b_0+b_1e^{xk+\omega t}\right)}$$

$$v(x,t)=\left(\frac{\left(\begin{array}{l} \dfrac{-(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)b_{-1}}{6(nb+d)}e^{-xk+\omega t}-\dfrac{(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)b_0}{6(nb+d)} \\ -\dfrac{b_1(3cn^2d-n^2\omega^2+9cnd-3n\omega^2+6cd-2\omega^2)}{6(nb+d)}e^{xk+\omega t} \end{array}\right)}{\left(b_{-1}e^{-xk+\omega t}+b_0+b_1e^{xk+\omega t}\right)}\right)^{1/n}$$

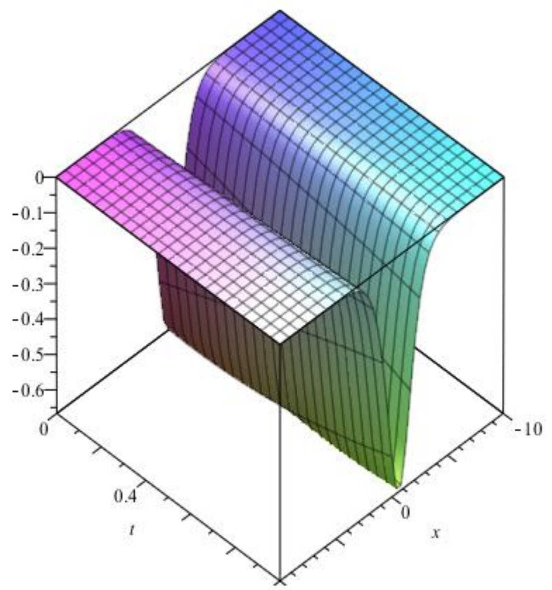

Figure 3. Graph of $v(x,t)$ for $a_0 = 1, b_1 = 1, k = 1, a = 1, b = 2, \omega = 1, d = 1, c = 1, n = 1$.

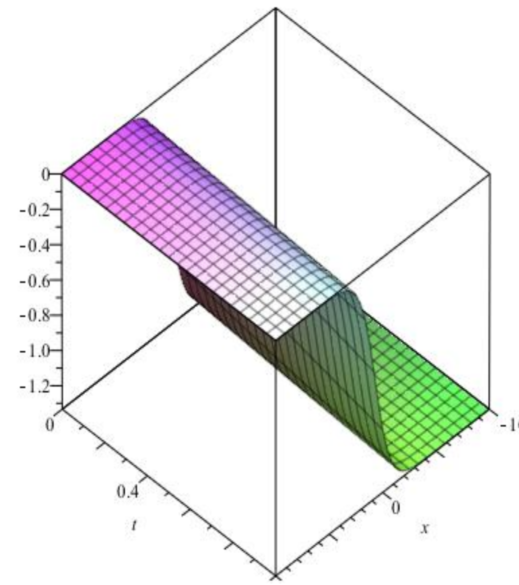

Figure 4. Graph of $v(x,t)$ for $b_1 = 1, b_2 = 1, k = 1, a = 1, b = 2, \omega = 1, d = 1, c = 1, n = 1$.

Case 3.1.2

If $p = c = 2,$ and $q = d = 1$, then equation (14) reduces to:

$$\phi(\eta) = \frac{a_2 \exp[2\eta] + a_1 \exp[\eta] + a_0 + a_{-1} \exp[-\eta]}{b_2 \exp[2\eta] + b_1 \exp[\eta] + a_0 + b_{-1} \exp[-\eta]}. \qquad (18)$$

Proceeding as before, we obtain the following solution sets:

1st solution set

$$\left\{ \begin{array}{l} k = \frac{\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}, a_{-1} = 0, a_0 = a_0, a_1 = \frac{-4b_1(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)}{9(nb+d)}, a_2 = 0, \\ b_{-1} = \frac{-27a_0^2(nb+d)^2}{4b_1(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)^2}, b_0 = 0, b_1 = b_1, \\ b_2 = \frac{4(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)b_1^2}{27a_0(nb+d)} \end{array} \right\}$$

We therefore obtained the following generalized solitary solution (Figure 3):

$$\phi(x,t) = \frac{\left(a_0 - \frac{4}{9(nb+d)}\left(b_1(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)\right)e^{\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}\right)}{\left(\begin{array}{l} -\frac{27a_0^2(nb+d)^2 e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}}{4b_1(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)^2} + b_1 e^{\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t} + \\ \frac{4}{27a_0(nb+d)}\left((3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)b_1^2 e^{\frac{2x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+2\omega t}\right) \end{array}\right)}$$

$$v(x,t) = \left(\frac{\left(a_0 - \frac{4}{9(nb+d)}\left(b_1(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)e^{\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}\right)\right)}{\left(\begin{array}{l} -\frac{27a_0^2(nb+d)^2 e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t}}{4b_1(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)^2} + b_1 e^{\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t} + \\ \frac{4}{27a_0(nb+d)}\left((3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)b_1^2 e^{\frac{2x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+2\omega t}\right) \end{array}\right)} \right)^{1/n}$$

2nd solution set

$$\left\{ \begin{array}{l} k = -\frac{\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}, a_{-1} = 0, a_0 = 0, a_1 = 0, a_2 = \frac{-b_2(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)}{3(nb+d)}, \\ b_{-1} = 0, b_0 = 0, b_1 = b_1, b_2 = b_2 \end{array} \right\}$$

We therefore obtained the following generalized solitary solution (Figure 4):

$$\phi(x,t) = \frac{-\frac{b_2}{3}(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)e^{2\left(-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t\right)}}{\left((nb+d)\left(b_1 e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t} + b_2 e^{2\left(-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t\right)}\right)\right)}$$

$$v(x,t) = \left(\frac{-\frac{b_2}{3}(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2)e^{2\left(-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t\right)}}{(nb+d)\left(b_1 e^{-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t} + b_2 e^{2\left(-\frac{x\sqrt{a\omega(3cd-\omega^2)}n}{a\omega}+\omega t\right)}\right)} \right)^{1/n}$$

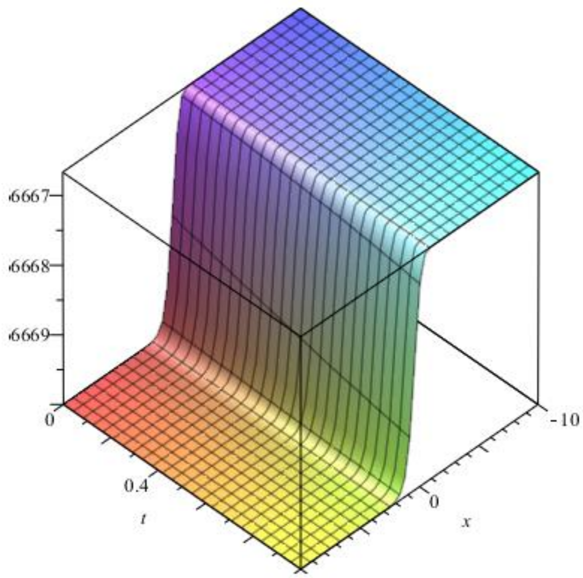

Figure 5. Graph of $v(x,t)$ for $b_1 = 1, b_2 = 0.1, b_{-1} = 1, b_0 = 2, k = 3, a = 0.1, b = 2, \omega = 1, d = 1, c = 1, n = 1\cdot$

3rd solution set

$$\left\{\begin{array}{l} a_{-1} = \dfrac{-\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_{-1}}{6(nb+d)}, a_0 = \dfrac{-\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_0}{6(nb+d)} \\ a_1 = \dfrac{-\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_1}{6(nb+d)}, a_2 = \dfrac{-\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_2}{6(nb+d)} \\ b_{-1} = b_{-1}, b_0 = b_0, b_1 = b_1, b_2 = b_2, k = k \end{array}\right\}$$

We therefore obtained the following generalized solitary solution (Figure 5):

$$\phi(x,t) = \frac{\left(\dfrac{-\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_{-1}e^{-kx+\omega t}}{6(nb+d)} - \dfrac{\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_0}{6(nb+d)} - \dfrac{b_1\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)e^{xk+\omega t}}{6(nb+d)} - \dfrac{b_2\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)e^{2(xk+\omega t)}}{6(nb+d)}\right)}{\left(b_{-1}e^{-xk+\omega t} + b_0 + b_1e^{xk+\omega t} + b_2e^{2(xk+\omega t)}\right)}$$

$$v(x,t) = \left(\frac{\left(\dfrac{-\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_{-1}e^{-xk+\omega t}}{6(nb+d)} - \dfrac{\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)b_0}{6(nb+d)} - \dfrac{b_1\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)e^{xk+\omega t}}{6(nb+d)} - \dfrac{b_2\left(3cn^2d - n^2\omega^2 + 9cnd - 3n\omega^2 + 6cd - 2\omega^2\right)e^{2(xk+\omega t)}}{6(nb+d)}\right)}{\left(b_{-1}e^{-xk+\omega t} + b_0 + b_1e^{xk+\omega t} + b_2e^{2(xk+\omega t)}\right)}\right)^{1/n}$$

In both cases, for different choices of c, p, d and q we get the same soliton solutions which clearly illustrate that the final solution does not strongly depends upon these parameters.

CONCLUSION

Exp-function method is applied to construct solitary solutions of the generalized Drinfel'd–Sokolov–Wilson system. The obtained results show that the applied method is very a convenient and powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. The Exp-function method can be also proposed for other nonlinear evolution equations with higher order nonlinearity. The reliability of the proposed algorithm is fully supported by the computational work, the subsequent results and graphical representations. It was observed that the exp-function method is very useful for finding solutions of a wide class of nonlinear problems.

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